Mathematical Cognition

A Case of Enculturation

Richard Menary

Most thinking about cognition proceeds on the assumption that we are born with our primary cognitive faculties intact and they simply need to mature, or be fine-tuned by learning mechanisms. Alternatively, a growing number of thinkers are aligning themselves to the view that a process of enculturation transforms our basic biological faculties. What evidence is there for this process of enculturation? A long period of development, learning-driven plasticity, and a cultural environment suffused with practices, symbols, and complex social interactions all speak in its favour. In this paper I will sketch in outline the commitments of the enculturated approach and then look at the case of mathematical cognition as a central example of enculturation. I will then defend the account against several objections.

Keywords
4E cognition | Ancient number system | Arithmetical cognition | Cognitive integration | Cultural inheritance | Discrete number system | Enculturation | Evolution of cognition | Evolutionary continuity | Mathematical cognition | Niche construction | Symbol systems | Symbolic thought

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1 Introduction

Since cognitive science took an ecological turn it has been casting around for new frameworks in which to conduct its main business: experimental research. Those who have taken the ecological turn are convinced that classical and brain-bound frameworks don't provide the necessary conceptual and experimental tools required to make sense of cognition in the wild (Hutchins 1995). A number of alternative frameworks have been proposed, with embodied cognition the most frequently adopted. The theoretical framework one uses to understand cognition has profound empirical consequences for scientific practice. For example, it influences what we consider to be the relevant phenomena of interest, what questions we ask about them, how we design and perform experiments, and how we interpret results (Beer 2000). The theoretical framework of classical computation, for example, approaches cognitive processing as a matter of input represented symbolically, which is then syntactically processed according to stored knowledge that the system has. It proposes a single "sandwich style" layer of cognitive processing, involving input, computation, and output (Hurley 2010).

The theoretical framework of CI (cognitive integration; Menary 2007) proposes something

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altogether different: multiple cognitive layers where neural, bodily, and environmental processes all conspire to complete cognitive tasks. Although the framework is unified by a dynamical systems description of the evolution of processing in the hybrid and multi-layered system, it recognises the novel contributions of the distinct processing profiles of the brain, body, and environment. Furthermore, the CI framework explains our cognitive capabilities for abstract symbolic thought by giving an evolutionary and developmental case for the plasticity of the brain in redeploying older neural circuits to new, culturally specific functions—such as reading, writing, and mathematics (Menary 2014). I call this a process of enculturation.

This paper seeks to outline the phylogenetic and ontogenetic conditions for the process of enculturation. It will take mathematical cognition, particularly the evolutionary basis for mathematical cognition, as a core example of enculturation. In so doing, I hope to have given an account of why enculturation exists, how it happens, and in what ways it can be defended against objections. In the first section I will explore the relationship of CI to cognition embodied, embedded, enacted, extended (4E) cognition and then explain why social and cultural practices are important to the process of enculturation. In the second section I will outline the core concepts required to make sense of enculturation: continuity, transformation, novelty, and uniqueness. The third section will introduce the example of mathematical cognition, moving from the evolutionary basis for numerosity and numerical cognition to the precise operations of mathematics. The fourth section will give an account of mathematical cognition as a case of enculturation. In the final section I outline two possible objections and respond to them.

2 Where does CI sit in the 4E landscape?

Traversing the 4E landscape one rises from the lowlands of weakly embodied and embedded cognitive science to the giddy heights of strong embodiment and embedding. Embodied cognition is the thesis that at least some of our cognitive states and processes are constituted by bodily processes that are not brain-bound. Embedded cognition is the thesis that our cognitive systems are located in and interact with the surrounding physical and social environment. Enactive and extended approaches to cognition inhabit the rarefied atmosphere of the strongly embodied and embedded peaks. However, there are important differences between enaction and extension and between those variants and CI. To determine where CI and enculturation sit in the 4E landscape, I will use a dimensional analysis I first introduced in Menary (2010).

> Embodied mind
> **Embodied mind weak**: the mind/brain is embodied (compatible with internalism/individualism Smart 1959; Stich 1983)
> **Embodied mind moderate**: some of our mental and cognitive processes and states depend[1] upon our non-neural body (Gallagher 2005; Gallese 2008)
> **Embodied mind strong**: some of our mental and cognitive processes and states are constituted by processes of the body acting in and on the environment (compatible with enactivism Varela et al. 1991, and CI Menary 2007)
>
> Embedded mind
> **Embedded mind weak**: All the perceptual inputs to and behavioural outputs from cognitive systems are found in the environment (compatible with internalism/individualism Adams & Aizawa 2008; Rupert 2009)
> **Embedded mind moderate**: Mental and cognitive states and processes are scaffolded or causally depend upon the environment (Sterelny 2003; Wheeler 2005)
> **Embedded mind strong**: Some mental and cognitive processes and states are integrated with environmental states and processes into a single system (compatible with extended mind Clark 2008, this collection; Menary 2007; Rowlands 2010)

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[1] Here we might take dependence simply to be a causal, and not a constitutive, relation. Perhaps my gesturing in a particular way causes my recalling a word.

Weakly embodied mind is just the old thesis that the mind is identical to the brain. One can be an individualist and hold to this form of embodiment, and I won't consider the implications of the view here. The work of some[2] embodied cognition researchers will fall under the moderate sense of embodiment. For example, those who attempt to show that concepts or word-meanings are causally dependent upon sensorimotor areas of the brain (Glenberg 2010; Gallese 2008) commit to a moderate sense of embodiment. The strong sense of embodiment focuses on how cognition is constituted by bodily interaction with the environment, and I shall focus on the discussion here. CI and enactivism occupy this region of the environment, but with different emphases on the nature of the interaction and the evolutionary continuity of simple and complex cognitive systems. CI also occupies the strongly-embedded region, but I shall deal with the relation between CI and cognitive extension in the next sub-section.

Enactivism (excluding its radical variant)[3] allows that even simple living systems are cognitive. Enactivists are committed to the continuity of life and mind and so they propose cognitive and even mental states and processes[4] for much simpler biological systems than would CI (Varela et al. 1991).[5] Whilst I am sympathetic with the commitment to continuity between simple cognitive systems and complex cognitive systems, it is questionable whether we should argue that simply being a living organism provides sufficient cognitive complexity for conscious experience and sense (or meaning) making.

CI does not require us to think that complex cognitive and mental phenomena, such as conscious experience, are shared by all living organisms whatever their complexity or simplicity. This is to assume that the properties of complex cognitive systems will be found even in very simple cognitive systems. According to CI, this gets things the wrong way round: there is a continuity from very simple systems that interact with their environments, by having mechanisms that track or detect salient features of their environments, to complex systems that have a wider range of cognitive capabilities (traits) including memory, inference, communication, problem solving, social cognition, and so on. By contrast a phylogeny of cognitive traits would show the distribution of those traits (across species) and help us to understand both the evolutionary pressures that produce more complex kinds of cognitive systems and the innovations that bring about new traits.[6]

CI provides a phylogenetic and ontogenetic basis for when bodily interactions are cognitive processes. Along with niche constructionists (Laland et al. 2000), CI maintains a phylogeny of hominid cognition in terms their active embodiment in a socially constructed cognitive niche. Ontogentically, neonates acquire cognitive abilities to create, maintain, and manipulate the shared cognitive niche, including tools, practices, and representational systems. Cognitive processing often involves these online bodily manipulations of the cognitive niche, sometimes as individuals and sometimes in collaboration with others. CI has a unique position on the 4E landscape, because it is the first framework to propose that the co-ordination dynamics of integrated cognitive systems are jointly orchestrated by biological and cultural functions. What, though, are the cultural functions in question?

2.1 Cognitive practices as cultural practices

Both CI and extended mind (EM) occupy the strong embedding region, but they do so in different ways. Here I will differentiate CI as a thesis of enculturation from Clark's organism-

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[2] One could look at a classic paper on mind/brain identity such as Smart (1959).

[3] See Thompson (2007) for an account of the life-mind continuity, Stewart et al. (2010) for a volume dedicated to enactivism, and Hutto & Myin (2013) for a self-proclaimed radical variant.

[4] See for example Barbaras (2010), which argues that to live is to have intentional consciousness of living.

[5] Interestingly, radical enactivsts appear to agree with CI on this issue; see Hutto & Myin (2013, p. 35). However, the radicals have a problem bridging the gap between basic cognitive processes and enculturated ones, since they think that meaning, or content, can only be present in a cognitive system when language and cultural scaffolding is present (Hutto & Myin 2013). That, of course, doesn't sit well with evolutionary continuity.

[6] See for example Sterelny's cognitive phylogeny in Sterelny (2003) and Godfrey-Smith's complexity thesis in Godfrey-Smith (1996). See MacLean et al. (2012) for an overview of the problems for a comparative phylogeny.

centred approach to EM. Cognitive integration is a model of how our minds become enculturated. Enculturation rests in the acquisition of cultural practices that are cognitive in nature. The practices transform our existing biological capacities, allowing us to complete cognitive tasks, in ways that our unenculturated brains and bodies will not allow. Cultural practices are patterns of action spread out across cultural groups (Roepstorff 2010; Hutchins 2011; Menary 2007, 2010, 2012). Cognitive practices[7] are enacted by creating and manipulating informational structures[8] in public space. This can be by creating shared linguistic content and developing it through dialogue, inference, and narrative; or it can be by bodily creating and manipulating environmental structures, which might be tools or public and shared representations (or a combination of both). Examples of linguistically mediated action include self-correction by use of spoken (or written) instructions, co-ordinating actions among a group, or solving a problem in a group by means of linguistic interaction. Examples of creating and manipulating public and shared representations include using a graph to represent quantitative relationships; using a diagram to represent the layout of a circuit or building; using a list to remember a sequence of actions; or to solve an equation, to mathematically model a domain, to make logical or causal connections between ideas, and so on. Practices can be combined into complex sequences of actions where the physical manipulation of tools is guided by spoken instructions, which are updated across group members. A simple example of a group brainstorming with one member writing out the answers would be an example of a complex of collaborative cognitive practices.[9] Cognitive practices are culturally endowed (bodily) manipulations of informational structures.

Practices govern how we deploy tools, writing systems, number systems, and other kinds of representational systems to complete cognitive tasks. These are not simply static vehicles that have contents; they are active components embedded in dynamical patterns of cultural practice. Practices are public, and they are also embodied and enacted.[10] We embody practices: they become the ways in which we act, think, and live. They structure our lifeways (although not exclusively).

CI does not deny that much thinking takes place offline in the brain, but it does take the online and interactive mode of thought to be adaptive. Again, this line of thought has precursors,[11] but CI, uniquely, takes interactive thought as a basic category,[12] which is then scaffolded by culturally evolved practices. Practices stabilise and govern interactive thought across a population of similar phenotypes. The stable patterns of action can then be inherited by the next generation, because the practices have become settled and are part of the developmental niche in which the minds of the next generation grow. Our brains co-adapted to the stable spread of practice and its role in ontogeny—resulting in the slow evolution of the cultural brain.

The focus upon practice and culture marks cognitive integration out from variants of extended cognition, such as Clark's organism-centred approach to extension (2008). Clark's organism centred approach takes the assembly of extended cognitive systems to be controlled by the discrete organism, and brain, at the centre of it. He thereby reduces the role of cultural practices in large or small groups of organ-

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[7] I don't mean to suggest that there can't be other effects of cognitive practices, but since practices are just the cultural formalisation of patterns of action across a population, or group, cognitive practices are tied directly to these patterns of action. I can't provide a detailed origin account for cognitive practices here, but see Menary (2007, Ch. 5) for an early attempt to do so. However, the account of mathematical cognition I give in the next two sections provides an example of how such an account would be likely to look.

[8] The primary cases I am thinking of are public systems of representation, including spoken language. However, I don't want to rule out cases involving tools, bodily gestures, artistic or bodily adornments, and the intelligent use of space and objects.

[9] For two very good overviews of collective or group cognition see Theiner (2013) and Huebner (2013).

[10] Jennifer Windt helpfully pointed out that practices can be thought of as public, because they are embodied and enacted. I think that this is just right: practices are patterns of action spread across a population. However, I am inclined to think that practices are not simply reducible to the bodily actions of individuals. Whilst doing long multiplication requires a bodily action of me, what I am doing cannot be described exclusively in terms of those bodily actions. The practice is a population, or group level phenomenon, not an individual one.

[11] The classical pragmatists, particularly Peirce and Dewey, held that thought was interactive. See Menary (2011) for a description of pragmatist approaches to thought, experience and the self.

[12] See Menary (2007, Ch. 5), where I make a detailed evolutionary case.

isms in the explanation of cognitive assembly. "Brains are special, and to assert this need mark no slippery-slope concession to good old-fashioned internalism as an account of mind. It is fully consistent with thinking (as I do) that Hutchins is absolutely right to stress the major role of transmitted cultural practices in setting the scene for various neurally-based processes of cognitive assembly" (Clark 2011, p. 458). On Clark's view, cultural practices only set the scene for the real work of integration to be done by the brain. Whilst it is arguable whether Clark's position is a return to "good old fashioned internalism," he certainly does not give cultural practices a central role in assembling and orchestrating cognitive systems.[13] Hutchins, by contrast, is committed to a full-blooded enculturated approach:

> [t]he ecological assemblies of human cognition make pervasive use of cultural products. They are always initially, and often subsequently, assembled on the spot in ongoing cultural practices. (2011, p. 445)

CI is the only variant of strong embedding (including EM) to explain the role of cultural practices in assembling integrated cognitive systems. Cognitive practices are inherited as part of the developmental niche and have profound transformative effects on our cognitive abilities. This leads us to the main concepts required to understand these transformations as a process of enculturation.

3 Enculturation: The main concepts

In this section I define and explain the main concepts required to understand enculturation, other than the already explored concepts of integration and practice. I will develop the concepts of evolutionary continuity, behavioural and neural plasticity, transformation and innovation, or novelty and uniqueness. In particular I will emphasise the phylogenetic and ontogenetic bases for modern human cognitive capacities.

3.1 Evolutionary continuity

The concept of evolutionary continuity results from the fact that evolution occurs gradually with complex structures evolving over many generations. Over long periods of time these gradual changes accumulate, resulting in large differences. Consequently, changes to a phenotype occur in slow cumulative steps over long periods of time and do not appear in a single mutational step. Evolutionary continuity demands that modern human minds evolved from earlier archaic variants. Doubtless modern minds differ from archaic minds in important respects, but these differences must have evolved over long periods of time, through slow cumulative mutational changes to the genotype. Even so, we should expect some of our archaic traits to remain, and for more modern variants to be built on top of them. One obvious example of this is the evolution of the human brain.

The evolution of the human brain can, to some extent, be seen in the gradual increase of cranial capacity, but some of the most important changes have been in the reorganisation of cortical circuitry and interconnectivity (Hoffman 2014). Although the evolution of the human brain can be understood in terms of increasing encephalization and increased connectivity between brain regions, the human brain has essentially the same set of structures as any other primate brain.[14] Modern brains evolved from archaic brains and share the same evolutionary constraints as other primates: "the similarity in brain design among primates, including humans, indicates that brain systems among related species are internally constrained and that the primate brain could only evolve within the context of a limited number of potential forms" (Hoffman 2014, p. 5). Modern minds are still partly archaic.

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[13] If this is an accurate portrayal of Clark's position (and I have tried to carefully use his own words) then, despite his protestations to the contrary, it appears to be a return to internalism, at least for the most central and important cognitive processes. If the brain carrys out all the important cognitive operations, then Clark's position would be a moderate embedded cognition for core cognitive abilities and an extended approach only to some of the more peripheral cases.

[14] "Although species vary in the number of cortical areas they posses, and in the patterns of connections within and between areas, the structural organization of the primate neocortex is remarkably similar" (Hoffman 2014, p. 4).

It is important to think of evolutionary continuity as running from archaic to modern. We should try to avoid anthropomorphic tendencies to project modern cognitive capacities backwards into the hominin lineage or across to primate species. For example, humans are excellent social cognisers, but it does not follow from this that we should expect other primates to have a theory of mind.[15] The evolutionary pressures under which humans evolved and the capacities for complex social cognition might have been very different from those under which other primates evolved. Consequently, we should be searching for archaic precursors to modern cognitive capacities. For example, we might expect that given the increasing social pressures in hominid social groups there would be precursors to modern social cognition and that these precursors would have been adaptive solutions (Shultz et al. 2012). Modern human social cognition would then be an evolutionary consequence of increasing variation in the complexity of social organisation and interaction (Sterelny 2003).

I am committed to another sense of continuity: that between biology and culture. Culture is not, as a category, distinct from the biological. Although culture is sometimes thought of as floating free of our biological nature and sometimes as being highly constrained by it, I shall assume that genes and culture co-evolve[16] mutually, influencing and constraining one another. Therefore I shall accept no culture–biology dualism in this paper. Indeed I shall adopt a cultural inheritance model of cognitive evolution (of the niche construction kind). However, I shall always do so with archaic origins in mind. Archaic origins matter to cognitive evolution and they matter to the way our brains develop during the lifespan.[17]

In the "modern synthesis" there is only one line of inheritance, and that is genetic inheritance. More recently, biologists (Odling-Smee et al. 2003) have proposed that there are other lines of inheritance: ecological inheritance and cultural inheritance (Boyd & Richerson 2005). Many organisms construct the niche in which they live, mate, hunt, and die. Niche constructors modify the ancestral environment, and these modifications are bequeathed to the next generation. Modifications encompass physical alterations, such as living in mounds or constructing hives, as well as cultural artefacts, practices, and institutions. Over long periods these alterations to the niche can have profound effects on the phenotype. For example, the ubiquitous niche constructions of termites, burrows and mounds, have profoundly altered their morphology and behaviour (Turner 2000).

Humans are also ubiquitous niche-constructors. They physically alter their environment and they also epistemically, socially, and culturally engineer the environment (Sterelny 2003, 2010; Menary 2007). Humans are born into a highly structured cognitive niche that contains not only physical artefact, but also representational systems that embody knowledge (writing systems, number systems, etc.); skills and methods for training and teaching new skills (Menary & Kirchhoff 2014); and practices for manipulating tools and representations. Inherited cultural capital is a real and stable feature of the socio-cultural environment, including a great variety of knowledge systems, skills, and practices across a variety of domains of human action. As such, human cultural niches provide neonates with rich developmental niches. It is in these developmental niches that humans acquire cognitive practices.

Cognitive practices are products of cultural evolution, evolving over faster timescales than biological evolution. Writing systems, for example, are only thousands of years old; consequently, it is highly unlikely that there is a "reading gene" or even an innate specialised "reading module." This is important: cognitive capacities for reading and writing, mathematics, and other culturally recent forms of cognition could not be biological adaptations (that

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[15] Indeed, it is questionable whether humans deploy a theory of mind, or at least, perhaps they only do so on rare occasions (Hutto 2008; Andrews 2012). Andrews has also argued that we may share a number of "mind reading" strategies with other primates that don't involve theory of mind (2012).

[16] See below for a niche construction account of gene-culture coevolution. I favour such an account because it helps us to understand how a developmental niche could have cumulative downstream evolutionary effects on phenotypes (Sterelny 2003).

[17] They matter because they are part of the developmental biases that produce a robust phenotype.

evolved over long periods of time). The timescales for their evolution are too short. It follows that the capacity for culturally recent forms of cognition must be acquired through learning and training.

Although there are no innate specialized modules for these recent forms of cognition, cortical circuits with which we are endowed through evolution are transformed to perform new culturally recent cognitive functions, even though they evolved to perform different functions. Recent cognitive innovations aside, there are good reasons to expect that evolution has driven us to think by interacting with the environment and that this is adaptive (Sterelny 2003 2012; Menary 2007; Wheeler & Clark 2008). However, it is the scaffolding of cultural practices that orchestrates the interactions—as in the case of written language and mathematics.

Structured socio-cultural niches have had profound evolutionary consequences in the hominin lineage. Structured niches have co-evolved with human phenotypic and developmental plasticity. We have evolved to be a behaviourally plastic species (Sterelny 2012) as well as a cultural species. In this co-evolution we have developed all manner of skills, practices, and activities. Why, though, are we so peculiarly behaviourally plastic? One good answer to this question is that human behavioural and developmental plasticity is an adaptive response to the variability and contingency of the local environment (Finlayson 2009; Sterelny 2003, 2012; Davies 2012). This is an alternative to the view that we are adapted to a pleistocene hunting and gathering environment—a view relied upon by many evolutionary psychologists (Barkow et al. 1992).

Critical to a co-evolutionary account of cultural practices is the evolution of human plasticity. Given that there is such a variety of cultural activity, we need an account of human evolution that will allow for variability in human behaviour. Second, we need a model that explains how innovations in our cultural niche are inherited and propagated, leading to changes in behaviour over time. The niche construction model explains how both of these causal factors could come into play. In the subsections below, I outline the importance of behavioural and neural plasticity, the concept of transformation, and those of novelty and uniqueness.

3.2 Behavioural and neural plasticity

In evolutionary terms, humans are capable of developing a wide range of skills that allow them to cope with a wide variety of environments (and their contingencies). For example, even where skills are (broadly) of the same type, such as hunting, they will vary in how they cope with the differences in local environments—think of the differences in environments between Aboriginal hunters in the Pilbara desert, hunter-gatherers in the Central American rainforests, and Inuit seal-hunters (Sterelny 2003, p. 167).

Development is extended in modern humans relative to other species. Humans take a long time to learn how to walk and talk, and much, much longer to develop fine-grained manual and cognitive skills such as reading and writing. Other primates have much faster developmental timescales. While this might make humans more dependent on their caregivers for longer, it also allows them to refine skills and acquire a greater array of them before entering adulthood.

Through cultural inheritance, knowledge, skills, and artefacts are passed on to the next generation, but learning environments and learning techniques are also passed on so that the next generation can acquire and be transformed by the inherited cultural capital. This last point is important for our purposes, because developmentally plastic humans need scaffolded learning environments in which to develop.[18]

How, though, are we capable of acquiring these new cultural capacities in development? Through neural plasticity. Rather than the process of synaptogenesis or lesion-induced plasticity,[19] the kind of plasticity I will discuss here is

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[18] If the cognitive abilities for manipulating artefacts and representations are not innate, then a scaffolded learning environment helps to explain how we acquire them.

[19] Many neurological studies of plasticity focus on synaptogenesis, the florid growth of grey matter and then the consequent pruning, or the

what I call learning driven plasticity (see Menary 2014). Learning driven plasticity (LDP) can result in both structural and functional changes in the brain. Structurally, LDP can result in new connections between existing cortical circuits. Functionally, LDP can result in new representational capacities (the ability to represent public symbolic representations such as alphabets and numerals) and new cognitive abilities, such as mathematics,[20] reading, and writing (Dehaene 2009; Ansari 2012). It should come as no surprise that learning drives structural and functional changes in the brain, given the extended developmental period in humans and the late development of the cortex (Thatcher 1991). The brain changes, not just because of maturation, but also because of learning:

> [w]hen children learn to read, they return from school 'literally changed'. Their brains will never be the same again. (Dehaene 2009, p. 210)

Famously, Dehaene argues that a region of the occipito-temporal junction (which he calls the VWFA, visual word form area) that is part of a wider network for recognising faces, objects, and even abstract shapes (such as chequer patterns), alters its function to recognise written symbols in alphabets and even logographic scripts such as kanji (Dehaene 2009). This is due to the plasticity of that area of the brain, where the functional shift is due to scaffolded learning.[21] "Scanning of 'ex-illiterate' adults who learned to read during adulthood has demonstrated that the VWFA is highly plastic, even in adults, and quickly enhances its response to letter strings as soon as the rudiments of reading are in place" (Dehaene & Cohen 2011, p. 259). Even those who are not convinced that a specialised region for "word recognition" is acquired once we learn to read admit that the occipito-temporal junction is part of a reading and writing circuit (e.g., Price & Devlin 2011).

We have evolved to be phenotypically and developmentally plastic. This is in no small part due to the plasticity of our brains. Our developmentally plastic brains exhibit learning-driven plasticity. When the brain is coupled to a highly scaffolded learning environment it is profoundly transformed, structurally and functionally, and consequently we are cognitively transformed in the profoundest way.

3.3 Transformation

The transformation thesis can be given a simple formulation: cognitive transformations occur when the development of the cognitive capacities of an individual are sculpted by the cultural and social niche of that individual. Cognitive transformations result from our evolved plasticity and scaffolded learning in the developmental niche. In the previous sub-sections an account was given of the effects of cultural inheritance and niche construction on hominid evolution. The result is phenotypic plasticity, and in the cognitive case the co-evolution of neural plasticity and scaffolded learning. However, the point of the transformation thesis is to drill down into the process of acquiring knowledge, skills, and cognitive abilities via learning-driven plasticity and scaffolded learning. It does this by showing how transformations are a result of the role of cognitive practices in development. Practices structure the niche; they transform plastic brains via learning driven plasticity and result in new cognitive abilities.

During the learning and training of a skill, such as flaking an arrowhead, or a shot in tennis or cricket, we are guided by the norms for the correct actions that make up the skilled practice. A parallel case can be made for cognitive abilities such as mathematics. The neophyte mathematician gains mastery over the cognitive norms[22] by which numerals, operators, and other symbols are created and manipulated. Vygotsky expresses this in the claim that children, "master the rules in accordance with which ex-

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synaptic death of many of those neurons in the so-called critical period of childhood. There are a large number of studies of neural damage, often by stroke or injury, where cortical circuitry becomes damaged and its function impaired, but where other areas of the cortex can take on the impaired function. (See Huttenlocher 2002 for an overview.)

[20] I will be defending an account of mathematical cognition in section 4.

[21] See Menary (2014) for a discussion of plasticity and the VWFA.

[22] For an account of cognitive norms see Menary (2007), Chapter 6.

ternal signs must be used” (Vygotsky 1981, pp. 184–185). Initially the child masters the creation and deployment of spoken linguistic signs (and later written signs) through the scaffolding of parents and caregivers. However, this process is not simply a matter of gaining new representations; it is also one of gaining new abilities.

Neophytes go through a process of dual-component transformation: they learn how to understand and deploy public symbolic representations and they learn how to create and manipulate inscriptions of those symbols in public space (Menary 2010). In so doing, they learn mathematical and linguistic concepts and they learn how to manipulate inscriptions to complete cognitive tasks. When learning the manipulative techniques, the first transformation is one of the sensory-motor abilities for creating and manipulating inscriptions: we learn algorithms like the partial products algorithm[23] and this is an example of the application of a cognitive practice. This is something we learn to do on the page and in the context of a learning environment, in public space, before we do it in our heads. Our capacities to think have been transformed, but in this instance they are capacities to manipulate inscriptions in public space. This is a way of showing that the transformation of our cognitive capacities has recognisably public features. This ought not to be a surprise, given that the cognitive niche is socially and culturally constructed and is structured by socio-cultural practices. Symbol systems, such as those for written language and mathematics, are not impermanent scaffolds that we shrug off in adulthood, but are permanent scaffolds that indelibly alter the architecture of cognition.[24]

The transformatory position is quite different from that held by Clark or Sterelny. In particular it holds that our basic cognitive capabilities are transformed in development and that the dual component transformation results in a distinct functional redeployment of neural circuitry and new abilities to bodily manipulate structures in public space. Cognitive tasks can be completed by manipulating written symbols in public space or by off-line strategies for completing algorithms, or a combination of both. This conclusion sits happily with the idea that thought is interactive and governed by practices.

The main difference between the position outlined here and Clark’s (e.g., 2008), is that Clark does not explain cognitive extension in terms of the transformation of basic cognitive resources during development in a socio-cultural niche (although he does acknowledge the importance of symbolically structured niches). Rather, he thinks that basic biological resources are not really transformed but simply dovetail to external symbols (Clark 2008, 2011). Sterelny (2010) concentrates on cognitive scaffolding, but does not think that the manipulation of symbols in public space is constitutive of cognitive processing. The enculturated approach of CI answers questions that are problematic for both Clark and Sterelny:

1. How do we learn to complete cognitive tasks that require the manipulation of symbols in public space?
2. Assuming that cognitive processing criss-crosses between neural space and public space, how does it do this?

The first question is hard for Clark since he does not think that our basic cognitive resources get transformed, at least in the way that I have presented here. The second question is hard for Sterelny because he limits himself to a scaffolded view of cognition rather than an extended view. Consequently, manipulations of symbols in public space are not cognitive processes for Sterelny.[25]

CI as a process of enculturation requires a robust transformation thesis. A robust transformation thesis is warranted by phenotypic and neural plasticity, in particular by learning driven plasticity. Novel and unique public systems of representation drive the transformation of our existing cognitive abilities.

3.4 Novelty and uniqueness

Sometimes symbols and tools provide us with novel functions: they radically extend our cap-

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[23] I’ll look at this example in detail in section 5.

[24] I take this issue up again in section 4.1.

[25] Or they might be assuming that Sterelny does not care either way; in private communication Sterelny indicated that he does not think that boundary disputes are of much interest.

abilities in some sphere. Take the humble hand axe. Very crude hand tools have been discovered dating as far back as 2.6 mya (million years ago; Toth & Schick 2006), since then there has been evidence of a hominid capacity for cumulative cultural inheritance "which was ultimately to transform Homo sapiens into the richly cultural species we are today" (Whiten et al. 2011). However, the capacity for developing novel functions and transmitting them to the next generation with high fidelity appears to be a more recent innovation, as evidenced by the long periods of relative stability in technological development in the early hominids and archaic humans. It also appears to be an innovation unique to the homonin lineage (Whiten et al. 2011). The Oldowan period begins in the lower paleoloithic with Homo Habilis around 2.6 mya, being taken up by Homo Erectus and Ergaster and ending at about 1.8 mya (Lycett & Gowlett 2008). The tool types and process of manufacture remain consistent during this period, with some refinement and novelty (Lycett & Gowlett 2008), where the main tool types were choppers and scrapers or mode 1 tools (Semaw et al. 2003).

Homo Habilis is unique in that it is the first hominid to make tools that were made to endure and be re-usable (it is likely that earlier anthropocines used naturally-occurring objects as tools that were disposable; Jeffares 2010).

> Oldowan toolmaking involves the production of sharp-edged flakes by striking one stone (the core) with another (the hammerstone). Effective flake detachment minimally requires visuomotor coordination and evaluation of core morphology (e.g., angles, surfaces) so that forceful blows may reliably be directed to appropriate targets (Stout et al. 2008, p. 1940).

There is a clear transition to Achulean technology at around 1.7 mya with the appearance of Erectus/Ergaster. The main innovation for Achulean technology was the bifacial handaxe—a handheld cutting tool with two cutting sides. The real explosion in novelty occurs in the upper paleoloithic period, from 50,000 years ago (ya) to 10,000 ya (or to just before the advent of agriculture and the neolithic period), with genuine novelty in tool production and use and cultural diversification. In this period we begin to see evidence of art, including paintings and sculpture, fishing, jewellery, burial, evidence of musical activity, and all the hallmarks of behaviourally modern humans. It is in this period that the combination of inherited cultural capital, with phenotypic and learning-driven plasticity, complex social relations and language results in an explosion of cultural and behavioural diversity.

It is also in this period that we begin to find evidence of proto-numerical and writing systems as novel representational innovations. Simple tally notch systems on bone fragments have been dated to between 35,000 and 20,000 ya, and may have been used for a variety of purposes, the most obvious being to keep track of economic exchanges. However, it is far easier and more economical to keep track of larger amounts using a single symbol, rather than a one-to-one correspondence of marks with things.

The complex social and economic pressures that required tracking exchanges involving increasingly large numbers would be the kind of socio-economic pressures that produced symbolisation of quantity. Social and cultural pressures can drive evolutionary novelty, in this case symbolisation and uniqueness—symbolic representations are unique in both type and property, no other animal produces written symbols to represent concepts. Symbols have unique properties that allow for operations—addition, subtraction, multiplication, division, and so on that are much harder (if not unlikely) without them.

Early symbolic number systems date from between 3000–4000 BCE, but genuinely abstract symbol systems are even more recent—about 1000–2000 BCE. The invention of symbol systems is too recent to be a genetic endowment, but is inherited as cultural capital and acquired through high-fidelity social learning (which is in turn dependent upon neural plasticity).

The phylogeny of hominid tool-use is one of hard-won innovation and retention. Modern humans have developed high-fidelity modes of transmitting cultural capital vertically and horizontally. The socio-cultural pressures that led to humans innovating symbolic representational systems are unique and very recent. Fortunately, modern human minds are flexible enough to both innovate and reliably acquire those innovations in ontogeny.[26] This flexibility makes modern human minds unique, and in the case of mathematical cognition unique amongst all our primate relatives.

The next section outlines mathematical cognition as a case of enculturation, and there I will explore the example of mathematical cognition by deploying the concepts refined in the first two sections.

4 Numerical cognition

In this section I outline the phylogenetic basis of mathematical cognition. That basis is in our shared sense of quantity and our ability to estimate the size of small sets by making approximate judgements of the size of the set. This ancient endowment is the basis for our mathematical competence, but it is not all there is to mathematical cognition. This is because precise mathematics depends upon a very recent and acquired public system of exact and discrete mathematical thinking. The ancient system is analogue and approximate, but mathematics requires digital and discrete representations and exact operations. These are, of course, recent additions to inherited cognitive capital. I shall show why mathematical cognition requires our ancient capacity for numerosity and how it is constituted by cognitive practices—which transform our cognitive abilities, resulting in novel and unique modern human cognitive capacities. However, this transformation results in two partially overlapping systems—the approximate number system and the discrete number system —with the latter having unique properties acquired from cultural innovation. One of the puzzles is how it is possible to move from an inherited approximate system to an acquired exact system. The process of enculturation provides the mechanisms by which such a move takes place, from the ancient capacity for numerosity to development in a socio-cultural niche, and the orchestrating role of practices in the assembly of the cognitive systems responsible for mathematical cognition.

4.1 Numerosity in animals and humans

There is strong evidence to suggest that we have a basic analogical and non-linguistic capacity to recognise quantity and number. I think that there is overwhelming evidence for an ancient evolutionary capacity to discriminate cardinality, and to determine in an approximate way the quantity of membership of sets. It is obvious how this capacity, for only very small sets, would be beneficial for activities such as foraging, hunting, and so on.

Recent studies have revealed that the neural populations that code for number are distributed in the intraparietal sulcus (Dehaene & Cohen 2007). A growing number of studies show that both animals and humans possess a rudimentary numerical competence, which is an evolutionary endowment. For example, red-backed salamanders have been shown to choose the larger of two groups of live prey (Uller et al. 2003). Single neuron activation studies in rhesus monkeys (Nieder et al. 2006) discovered that individual neurons respond to changes in number when presented visually (and non-symbollically). These neurons are also located in the intraparietal sulci, indicating a probable cross-species homology. The neurons peak at the presentation of a specific quantity of dots, but then decrease as the numbers presented differ from the original. So a neuron that peaks at the presentation of two dots responds less to three or four dots. The further the numerical distance of the array of dots is from the magnitude to which the neuron is tuned, the lower the firing rate of the neuron. Therefore, the ancient capacity for numerosity is an approximate function, not a discrete one (DeCruz 2008).

This is not yet counting; counting is exact enumeration. Subitizing is the ability to immediately recognise the size, or number, of a small set—usually <4. Most animals subitize, rather than count. Infant humans also appear to be able to subitize (Rousselle & Noël 2008). This ancient or approximate number system (ANS)

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[26] This section has put together a case for the flexibility of modern minds and the ability to acquire cultural innovations quickly and easily in ontogeny.

is a non-linguistic continuous representation[27] of quantities above 4; Dehaene calls it the number sense (1997). Take the following example. Whilst it is easy enough to determine which of the following two boxes contains the larger number of dots without having to count them:

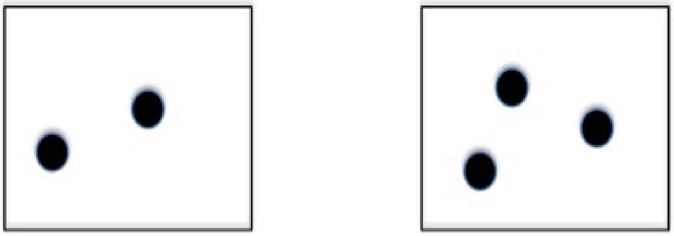

Figure 1: Subitizing or counting?

It is less easy to do so for the following (you will probably need to resort to counting):

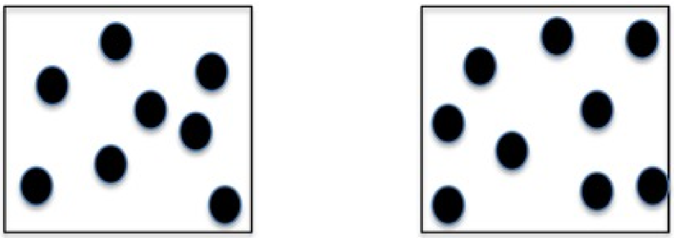

Figure 2: Subitizing or counting?

It is also possible to make estimations or approximate judgements of scale for numbers. Most people can quickly identify that 7 is larger than 3. Even for more complicated exact operations we can do this:

$34 + 47 = 268$ (is this right?)

We readily reject this result, because the proposed quantity is too distant from the operands of the addition (Dehaene 2001, p. 28).

$34 \text{ x } 47 = 1598$ (is this right?)

Approximation involving proximity and distance will not help here (unless you are very practised at mental multiplication), but you might resort to a multiplication algorithm (which might be routinized). It is clear that we have an ancient sense of quantity and are good at making judgements about more than and less than, but when it comes to precise and discrete quantities (particularly larger numbers) we need new capacities to be able to make judgements about operations on discrete numbers.

4.2 Two overlapping systems

The approximate numerical system is an analogue and approximate system for discriminating non-symbolic numerosities greater than 4, but the “representations” are approximate and noisy. The second system is acquired and concerns discrete symbolic and linguistic representation of individual numbers from our numeral system, including individual words for numbers. This system works with discrete, exact, symbolic representations of quantity and allows for the exact operations of arithmetic and mathematics. I will call this the discrete numerical system (DNS). There is disagreement about how much the two systems overlap. However, what is clear is that the internalisation of the public numeral system allows us to perform the kind of digital mathematical operations that are required for most arithmetic and mathematical operations (Nieder & Dehaene 2009, p. 197).

Dehaene and colleagues produced a series of experiments that demonstrate the separate functioning of the two systems. Russian–English bilinguals were taught a set of exact and approximate sums of two digit numbers in one of their languages (Dehaene et al. 1999, p. 970). Their tasks were split into giving exact answers to additions and giving an approximate answer to the addition task. The interesting result was that:

> [w]hen tested on trained exact addition problems, subjects performed faster in the teaching language than in the untrained language, whether they were trained in Russian or English. (Dehaene et al. 1999, p. 971)

This provided evidence that knowledge of arithmetic was being stored in a linguistic format,

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[27] The appearance of the word representation here need not raise concerns; these are not representations with propositional contents and truth conditions. They are not symbolic and are not molecular constituents that can be combined to make more complex representations.

and that there was a switching cost between the trained and untrained languages. By contrast, there was equivalent performance in the approximation task, and no switching cost between the trained and untrained languages. Dehaene et al. conclude that this provides "evidence that the knowledge acquired by exposure to approximate problems was stored in a language-independent form" (1999, p. 971).

This leads us to the conclusion that there are two overlapping, but not identical, systems for mathematical cognition. The first is the ancient and approximate system, the second is a relatively new and acquired system for discrete and digital representations and operations. As Dehaene & Cohen put it:

> The model that emerges suggests that we all possess an intuition about numbers and a sense of quantities and of their additive nature. Upon this central kernel of understanding are grafted the arbitrary cultural symbols of words and numbers […]. The arithmetic intuition that we inherit through evolution is continuous and approximate. The learning of words and numbers makes it digital and precise. Symbols give us access to sequential algorithms for exact calculations. (2007, p. 41)

The two systems are overlapping but not identical because they have quite different properties. First, the ancient system is part of our phylogeny, whereas the discrete system is an acquired set of capacities in ontogeny. Second, the ancient system is analogue and approximate, whereas the discrete system is digital and exact. Third, the discrete system operates on symbols that don't map directly on to the ancient system.

When we consider very large numbers, such as 10,000,000, there is no obvious analogue in the ANS. Consequently, large or exotic numbers and operations on them do not map onto existing cortical circuitry for numerosity. Lyons et al. (2012) call this phenomenon "symbolic estrangement". Symbols become estranged through a process of symbol-to-symbol mappings, rather than symbol-to-approximate-quantity mappings (Lyons et al. 2012, p. 635).

However, there appears to be a point of contention here: Dehaene expects there to be a more or less direct mapping of symbols to quantities (e.g., the mental number line). If symbolic estrangement does happen, then this would appear to be mistaken. Lyons, Ansari and Beilock propose a developmental resolution of this apparent disagreement. Children may start out in the acquisition of discrete number systems by a mapping to an existing approximate neural coding of quantity, but as the system matures and symbols become abstracted from the ancient system, the mature system splits into two (related but not entirely overlapping) systems: neural circuitry in the DNS tunes for discrete symbols,[28] whereas circuitry in the ANS tunes for approximate quantities, such that discrete symbols do not map directly onto approximate quantities. E.g., 10,000,000. The DNS has properties that are unique.

In the next section I return to the question of the role of practices in assembling the DNS.

5 Mathematical practices

The DNS is dependent upon mathematical practices, systems of number and algorithms for performing mathematical operations, complex mathematical concepts such as sets, functions, and so on. None of these practices, representations, or concepts are innate, and no one seriously thinks that they are. They are culturally inherited and acquired in the right learning niche with experts willing to teach. These new abilities are continuous with our cognitive phylogeny. How, though, can we put the whole package together? This section does that job.

5.1 Cognitive practices and the development of mathematical competence

Mathematics and writing systems are examples of culturally evolved symbol systems that are deployed to complete complex cognitive tasks. These systems are structured by rules and

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[28] There is evidence of narrower tuning curves for Arabic numerals in the left intraparietal sulcus (Ansari 2008).

norms, but they are deployed as practices: patterns of action spread out across a population. In this case cognitive agents must gain mastery over the symbols, including numerals and operators, as well as the rules for their combination. However, they must also learn how to write and manipulate the symbols according to those rules in order to produce the right products—and this is proceduralised.

There may be more than one way of achieving a solution to the task. One can multiply by the partial products algorithm, or one can use the lattice/grid method or a number of others that have been developed by different cultures using different numerical systems. However, they all involve the same set of features: symbols, rules, operators, spatial configuration, and products, and they jointly constitute a practice for manipulating the symbols to complete mathematical problems. The practices are novel and unique to humans.

The methods apply equally to their off-line equivalents, so in the page-based version of the partial products algorithm we perform the multiplications from right to left and write down their products in rows, carrying numbers where necessary. In the off-line version we can perform the same operations on imagined numerals, multiplying numbers along the line and carrying any numbers as required. It is cognitively taxing to hold the products of the multiplications constant in working memory, though some people can train themselves to become quite good at it. Most people learn off-line multiplication by performing shortcuts; if I want to work out what 25 x 7 is, I just add 25 together 7 times.

On-line methods can change even within the same arithmetical systems, so the partial products algorithm works like this:

	23	
x	11	
	23	(1x3 and 1x2)
+	230	(carry 0, 1x2 and 1x3)
	253	(add products together)

However there is an equivalent algorithm that works like this:

	23	
x	11	
	200	(10 x 20)
	30	(10 x 3)
+	23	(1 x23)
	253	(add products together)

The algorithms may differ, but they still involve the practice of spatially arranging the numerals, and performing operations on them and deriving a product, by performing the staged manipulations on the page. It appears then to matter how we manipulate symbols in public space, but is there any empirical evidence for this conclusion?

CI predicts that it matters how symbols are spatially arranged when they are being manipulated. Landy & Goldstone (2007) found that college-level algebraists could be induced to make errors by altering the layout of numbers that they were to manipulate. They did this by altering the spacing of the equations:

$$F+z \ * \ t+b = z+f \ * \ b+t$$

Although minor, the extra spacing was enough to induce errors. It matters how the symbols are spatially laid out, for this layout is the basis of how we manipulate those symbols. In this case the artificial visual groups created by the irregular spacing affected the judgement of the validity of the equation. If the visual groupings were inconsistent with valid operator precedence then they negatively affected the judgement.[29]

Landy & Goldstone's work provides evidence that expert algebraists are practised at symbolic reasoning achieved via the perception and manipulation of physical notations (2007; Landy et al. 2014). Rather than an internal system of abstract symbols and rules for their combination (i.e., a language of thought), the system is composed of perceptual-motor systems and the manipulations of numerals. They are careful to say that the manipulations must conform to the abstract norms of algebra. Dutilh Novaes (2013) takes this to be evidence that mathematical competence is constituted by the

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[29] In algebra multiplications are made before additions. E.g., 5+2*6 = 17 (not 42).

capacity to manipulate inscriptions of mathematical equations. This fits very well with the CI approach.

Despite some interesting lacunae (savants and blind mathematicians), most mathematicians learn to manipulate numerals and other mathematical symbols on the page, and they continue to do so throughout their mature cognitive lives. Landy and Goldstone's evidence supports the thesis that mathematical competence is constituted, in part, by our capacity to manipulate symbols in public space; that competence is, properly, a matter of interaction.

5.2 Continuity and transformation

We have seen that there is an ancient evolutionary endowment for numerosity—an analogue and approximate system. This system is found in other primates and other species. It provides both the phylogenetic basis of mathematical cognition and the initial constraints for the development of the DNS. The DNS did not spring sui generis into the world. It did so because of a heady mixture of socio-cultural pressures, phenotypic and neural plasticity, social learning strategies, and cultural inheritance. These are the conditions for the scaffolding of the ANS, transforming our basic biological capacities into the DNS.

New cultural functions, discrete mathematical functions, and the practices for manipulating inscriptions transform existing circuitry in the brain. Once we learn how to recognise, understand, and manipulate mathematical symbols our brains undergo a profound transformation. There is a reproducible circuit for mathematical cognition involving a bi-lateral parietal based approximate estimation; a left lateralised verbal framework for arithmetic concepts (e.g., number words); and a occipito-temporal based symbol recognition system (e.g., Arabic numerals). The system also incorporates visual-motor systems for writing (manipulating, or pushing) symbols in public space.

A further important aspect of transformation is symbolic estrangement. As the DNS matures it becomes more abstract and less directly mapped onto the approximate functions of the ANS. Interestingly, at the same time expert mathematicians become reliant upon visual-motor capacities for manipulating inscriptions. Transformation depends upon the novelty and uniqueness of mathematical symbols and practices.

5.3 Novelty and uniqueness

Symbolic number systems and sequential algorithms allow for mathematical and cognitive novelty. Once we have a public system, all manner of exotic numbers and operations can be discovered:[30] negative numbers, square roots, zero, sets, and so on. Its importance lies in the ability to perform computations that cannot be performed by ancient neural functions for numerosity. For example, the neural circuits responsible for numerosity cannot (on their own) represent -3 or $\sqrt{54}$, and yet this is simply represented in terms of public mathematical symbols (DeCruz 2008). This is because the symbolic representations are novel and unique. Initially, novelty results from the pressures of increasing social and economic complexity. Small roaming bands of foragers do not need to develop symbolic number systems; post-agricultural Neolithic societies settled in villages and towns do. A further issue is how novelty comes about from the ability to abstractly combine symbols and functions that apply to the symbols. I don't propose to try to answer that question here; however, we might think of this as a curiosity- and creativity-driven processes. Given uniquely human behavioural and neural plasticity and socio-cultural complexity we might expect an increasing drive towards cognitive innovation. This has certainly been the story of recent cultural evolution in modern human societies.

This concludes the discussion of mathematical cognition as enculturation. Now I turn to the objections.

6 The incredible shrinking system

Why not just shrink the cognitive system to brain-based systems? Is there a way to bridge

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[30] I will not address the issue of what discovery amounts to here and will remain neutral on whether discovery reveals a platonic mathematical system or simply the logical relations between concepts.

the impasse between moderate and strong embedding? One argument concerns whether it makes any difference to cognitive science to consider, for example, the manipulation of public symbols to be cognitive processes (Sprevak 2010). Ultimately, to give a decisive answer to that question we would need to change our conception of cognitive processes to on-going dynamical interactions with the environment that loop through brain, body, and environment. However, weak and moderate embedded approaches do not work with such a conception of cognitive process; they work with an input-process-output style sandwich model, where processes supervene on bodily states and processes. For them, there is no reason to accept strong embedding, and much of the discussion has been based around thought experiments or abstract definitions rather than concrete examples.

However, even on a scaffolded view of cognition we can't deny the difference-making role the manipulations of symbols make to the completion of cognitive tasks. Manipulating public symbols is unique; there is a difference between internalised strategies for completing mathematical tasks and strategies for manipulating mathematical inscriptions. Our cognitive capacities cannot cope with long sequences of complex symbols and operations on them. This is why we must learn strategies and methods for writing out proofs. Symbol manipulation makes a unique difference to our ability to complete mathematical tasks, and we cannot simply ignore their role. If we take the approach of CI, then mathematical cognition is constituted by these bouts of symbol manipulation, and we cannot simply shrink the system back to the brain. The case for a strongly embedded approach to mathematical cognition depends upon the novelty and uniqueness of mathematical practices and dual component transformations. Our evolutionary endowments of numerosity are not up to the task of exact symbolic arithmetic and mathematics. Without symbolic number systems and sequential algorithms there would be no mathematical innovation. Mathematical innovation includes representational novelty: negative numbers, square roots, zero, etc., but also novel functions: multiplication, division, etc. Novelty comes about from the ability to abstractly combine symbols and functions that apply to the symbols.

Uniquely, symbols represent quantities discretely, but there is also the unique human capacity of manipulating symbols in public space. We learn to manipulate symbols in public space and we continue to do so when completing cognitive tasks.

The entire system of mathematics is not contained in a single brain. Symbol systems are public systems of representations and practices for their manipulation. Mathematical practices are part of the niche that we inherit—they are part of our cultural inheritance.

6.1 Impermanent scaffolds?

Another objection concerns the impermenance of the scaffolding required for mathematical cognition. Once we have internalised the scaffolding of symbolic number systems, we have no further need for it, except for communication purposes. This claim would be proven if we did not continue to manipulate numerals when completing cognitive tasks. Even if we think that transformation only results in new internal representational resources, and that this just amounts to moderate embedding/scaffolding, we must also concede that most mathematics is conducted on the page.

Scaffolding theorists, like Sterelny, can endorse this idea; indeed they can agree with the bulk of the framework provided by CI whilst avoiding the constitutive claim. What they cannot do is deny that mathematical practice and the manipulation of physically laid-out symbols on the page is a difference maker for mathematical cognition. If you remove it, the ability to complete mathematical tasks drops considerably. To do so is to fly in the face of the empirical evidence from psychology (Landy & Goldstone 2007) and cognitive neuroscience (Dehaene & Cohen 2007; Ansari 2012). Consequently, it is clear that cognitive practices transform our mathematical abilities, lending weight to the CI approach.

The case I have presented in this paper is that symbols are not simply impermanent scaffolds, they are permanent scaffolds. They become part of the architecture of cognition (and not simply through internalisation). Mastery of symbol systems results in changes to cortical circuitry, altering function and sensitivity to a new, public, representational system. However, it also results in new sensori-motor capacities for manipulating symbols in public space. The case can be made in terms of what a symbol system is:

A symbol is a physical mark (or trace), either in physical space, or as a digital trace. Symbol systems contain rules and practices for interpreting symbols, for combining them, and for ordering and manipulating them. A large body of often tacit practices for interpreting and manipulating symbols is acquired. Scaffolding is not simply an amodal symbol with an abstract designation that needs to be learnt (or mapped onto some innate symbol); scaffolding is also how the symbols are physically arranged, how symbols are pushed from one place to the next in a regular fashion. Finally, scaffolding is also how we use our own bodies, eyes, ears, and hands to create and manipulate symbols.

7 Conclusion

I have presented a case for CI as a process of enculturation, with mathematical cognition as an example of the process of enculturation at work. I began by laying out the 4E landscape and locating CI within it, relative to enactivism and EM. In particular I showed how CI shares the interactive stance of enactivism and the constitutive stance of EM, but how it also differs from these. The main difference between CI and enactivism is that CI does not equate life and mind in the way that enactivism does. The main difference between CI and EM is that CI takes cultural practices to play a central role in the assembly of cognitive systems, whereas EM does not.

I then went on to outline the central concepts required to make sense of enculturation. The CI framework embraces both evolutionary continuity and transformation of existing cognitive circuitry in development. Our modern minds are built on archaic precursors by slow incremental changes. However, modern humans are behaviourally plastic and scaffolded learning drives functional changes in our plastic brains. The developmental change from the ANS to the DNS is an example of how learning-driven changes to cortical function result in new abilities, but this would not happen without the novelty and uniqueness of mathematical symbols and the practices for manipulating them.

I also countered two standard objections: impermanence and shrinkage. The defence of CI rested on the novelty and uniqueness of mathematical practices and symbols.

If the CI framework is on the right track, then human cognitive evolution has resulted in minds that are flexible and interactive. Furthermore, cultural evolution has resulted in written symbol systems and practices for manipulating symbols that can be acquired (in development) by minds like ours. The uniqueness of modern human minds lies in their capacity for transformation.

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